INTRODUCTION & RECTILINEAR KINEMATICS: CONTINUOUS MOTION (Sections 12.1 - 12.2)

Today's Objectives:
Students will be able to find the kinematic quantities (position, displacement, velocity, and acceleration) of a particle traveling along a straight path.

In-Class Activities:
- Check homework, if any
- Reading quiz
- Applications
- Relations between s(t), v(t), and a(t) for general rectilinear motion
- Relations between s(t), v(t), and a(t) when acceleration is constant
- Concept quiz
- Group problem solving
- Attention quiz

READING QUIZ

1. In dynamics, a particle is assumed to have ________.
   A) both translation and rotational motions
   B) only a mass
   C) a mass but the size and shape cannot be neglected
   D) no mass or size or shape, it is just a point

2. The average speed is defined as ________.
   A) $\Delta r/\Delta t$
   B) $\Delta s/\Delta t$
   C) $s_r/\Delta t$
   D) None of the above.
Applications

The motion of large objects, such as rockets, airplanes, or cars, can often be analyzed as if they were particles.

Why?

If we measure the altitude of this rocket as a function of time, how can we determine its velocity and acceleration?

Applications (continued)

A train travels along a straight length of track.

Can we treat the train as a particle?

If the train accelerates at a constant rate, how can we determine its position and velocity at some instant?
An Overview of Mechanics

Mechanics: the study of how bodies react to forces acting on them

Statics: the study of bodies in equilibrium

Dynamics:
1. Kinematics – concerned with the geometric aspects of motion
2. Kinetics - concerned with the forces causing the motion

POSITION AND DISPLACEMENT

A particle travels along a straight-line path defined by the coordinate axis s.

The position of the particle at any instant, relative to the origin, O, is defined by the position vector \( \mathbf{r} \), or the scalar \( s \). Scalar \( s \) can be positive or negative. Typical units for \( \mathbf{r} \) and \( s \) are meters (m) or feet (ft).

The displacement of the particle is defined as its change in position.

Vector form: \( \Delta \mathbf{r} = \mathbf{r}' - \mathbf{r} \)  
Scalar form: \( \Delta s = s' - s \)

The total distance traveled by the particle, \( s_T \), is a positive scalar that represents the total length of the path over which the particle travels.
VELOCITY

Velocity is a measure of the rate of change in the position of a particle. It is a vector quantity (it has both magnitude and direction). The magnitude of the velocity is called speed, with units of m/s or ft/s.

\[ v_{avg} = \frac{\Delta r}{\Delta t} \]

The average velocity of a particle during a time interval \( \Delta t \) is

The instantaneous velocity is the time-derivative of position.

\[ v = \frac{dr}{dt} \]

Speed is the magnitude of velocity: \( v = \frac{ds}{dt} \)

Average speed is the total distance traveled divided by elapsed time:

\[ (v_{sp})_{avg} = \frac{s_T}{\Delta t} \]

ACCELERATION

Acceleration is the rate of change in the velocity of a particle. It is a vector quantity. Typical unit is m/s².

The instantaneous acceleration is the time derivative of velocity.

\[ a = \frac{dv}{dt} \]

Vector form: \( a = \frac{dv}{dt} \)

Scalar form: \( a = \frac{dv}{dt} = \frac{d^2s}{dt^2} \)

Acceleration can be positive (speed increasing) or negative (speed decreasing).

As the book indicates, the derivative equations for velocity and acceleration can be manipulated to get

\[ a \, ds = v \, dv \]
SUMMARY OF KINEMATIC RELATIONS:
RECTILINEAR MOTION

• Differentiate position to get velocity and acceleration.

\[ v = \frac{ds}{dt} \; \text{or} \; a = \frac{dv}{dt} = \frac{v}{ds} \]

• Integrate acceleration for velocity and position.

Velocity:
\[ \int_{v_o}^{v} dv = \int_{t_o}^{t} a \, dt \; \text{or} \; \int_{v_o}^{\dot{v}} v \, dv = \int_{s_o}^{s} a \, ds \]

Position:
\[ \int_{s_o}^{s} ds = \int_{t_o}^{t} v \, dt \]

• Note that \( s_o \) and \( v_o \) represent the initial position and velocity of the particle at \( t = 0 \).

CONSTANT ACCELERATION

The three kinematic equations can be integrated for the special case when acceleration is constant (\( a = a_c \)) to obtain very useful equations. A common example of constant acceleration is gravity; i.e., a body freely falling toward earth. In this case, \( a_c = g = 9.81 \text{ m/s}^2 \) downward. These equations are:

\[ \int_{v_o}^{v} dv = \int_{t_o}^{t} a_c \, dt \; \text{yields} \; \dot{v} = v_o + a_c t \]

\[ \int_{s_o}^{s} ds = \int_{t_o}^{t} v \, dt \; \text{yields} \; s = s_o + v_o t + \frac{1}{2} a_c t^2 \]

\[ \int_{v_o}^{\dot{v}} v \, dv = \int_{s_o}^{s} a_c \, ds \; \text{yields} \; \dot{v}^2 = (v_o)^2 + 2a_c (s - s_o) \]
EXAMPLE

**Given:** A motorcyclist travels along a straight road at a speed of 27 m/s. When the brakes are applied, the motorcycle decelerates at a rate of -6t m/s².

**Find:** The distance the motorcycle travels before it stops.

**Plan:** Establish the positive coordinate s in the direction the motorcycle is traveling. Since the acceleration is given as a function of time, integrate it once to calculate the velocity and again to calculate the position.

**Solution:**

1) Integrate acceleration to determine the velocity.
   \[ a = \frac{dv}{dt} \Rightarrow dv = a \, dt \Rightarrow \int_{v_i}^{v} dv = \int_{0}^{t} (-6t) \, dt \]
   \[ v - v_i = -3t^2 \Rightarrow v = -3t^2 + v_i \]

2) We can now determine the amount of time required for the motorcycle to stop (v = 0). Use \( v_i = 27 \) m/s.
   \( 0 = -3t^2 + 27 \Rightarrow t = 3 \) s

3) Now calculate the distance traveled in 3s by integrating the velocity using \( s_0 = 0 \):
   \[ v = \frac{ds}{dt} \Rightarrow ds = v \, dt \Rightarrow \int_{s_0}^{s} ds = \int_{0}^{3} (-3t^2 + v_i) \, dt \]
   \[ s - s_0 = -t^3 + v_i t \]
   \[ s - 0 = (3)^3 + (27)(3) \Rightarrow s = 54 \text{ m} \]
CONCEPT QUIZ

1. A particle moves along a horizontal path with its velocity varying with time as shown. The average acceleration of the particle is _________.
   A) 0.4 m/s²   B) 0.4 m/s²
   C) 1.6 m/s²   D) 1.6 m/s²

2. A particle has an initial velocity of 30 m/s to the left. If it then passes through the same location 5 seconds later with a velocity of 50 m/s to the right, the average velocity of the particle during the 5 s time interval is _________.
   A) 10 m/s   B) 40 m/s
   C) 16 m/s   D) 0 m/s

GROUP PROBLEM SOLVING

Given: Ball A is released from rest at a height of 12 m at the same time that ball B is thrown upward, 1.5 m from the ground. The balls pass one another at a height of 6 m.

Find: The speed at which ball B was thrown upward.

Plan: Both balls experience a constant downward acceleration of 9.81 m/s². Apply the formulas for constant acceleration, with $a_c = -9.81 \text{ m/s}^2$. 
GROUP PROBLEM SOLVING (continued)

Solution:

1) First consider ball A. With the origin defined at the ground, ball A is released from rest \((v_A)_o = 0\) at a height of 12 m \((s_A)_o = 12\) m. Calculate the time required for ball A to drop to 6 m \((s_A = 6\) m) using a position equation.

\[
s_A = (s_A)_o + (v_A)_ot + (1/2)a_vt^2
\]

\[
6\ m = 12\ m + (0)(t) + (1/2)(-9.81)(t^2) \quad \Rightarrow \quad t = 1.106\ s
\]

2) Now consider ball B. It is thrown upward from a height of 5 ft \((s_B)_o = 1.5\) m. It must reach a height of 6 m \((s_B = 6\) m) at the same time ball A reaches this height \((t = 1.106\ s)\). Apply the position equation again to ball B using \(t = 1.106s\).

\[
s_B = (s_B)_o + (v_B)_ot + (1/2)a_vt^2
\]

\[
6\ m = 1.5 + (v_B)_o(1.106) + (1/2)(-9.81)(1.106)^2
\]

\[
\Rightarrow (v_B)_o = 9.49\ m/s
\]

ATTENTION QUIZ

1. A particle has an initial velocity of 3 m/s to the left at \(s_0 = 0\) m. Determine its position when \(t = 3\) s if the acceleration is 2 m/s\(^2\) to the right.

A) 0.0 m \hspace{1cm} B) 6.0 m
C) 18.0 m \hspace{1cm} D) 9.0 m

2. A particle is moving with an initial velocity of \(v = 12\) m/s and constant acceleration of 3.78 m/s\(^2\) in the same direction as the velocity. Determine the distance the particle has traveled when the velocity reaches 30 m/s.

A) 50 m \hspace{1cm} B) 100 m
C) 150 m \hspace{1cm} D) 200 m
RECTILINEAR KINEMATICS: ERRATIC MOTION
(Section 12.3)

Today’s Objectives:
Students will be able to determine position, velocity, and acceleration of a particle using graphs.

In-Class Activities:
• Check homework, if any
• Reading quiz
• Applications
• s-t, v-t, a-t, v-s, and a-s diagrams
• Concept quiz
• Group problem solving
• Attention quiz

READING QUIZ

1. The slope of a v-t graph at any instant represents instantaneous

   A) velocity.   B) acceleration.
   C) position.   D) jerk.

2. Displacement of a particle in a given time interval equals the area under the ___ graph during that time.

   A) a-t       B) a-s
   C) v-t       C) s-t
APPLICATION

In many experiments, a velocity versus position (v-s) profile is obtained.

If we have a v-s graph for the rocket sled, can we determine its acceleration at position \( s = 300 \) meters?

How?

GRAPHING

Graphing provides a good way to handle complex motions that would be difficult to describe with formulas. Graphs also provide a visual description of motion and reinforce the calculus concepts of differentiation and integration as used in dynamics.

The approach builds on the facts that slope and differentiation are linked and that integration can be thought of as finding the area under a curve.
S-T GRAPH

Plots of position vs. time can be used to find velocity vs. time curves. Finding the slope of the line tangent to the motion curve at any point is the velocity at that point (or \( v = \frac{ds}{dt} \)).

Therefore, the v-t graph can be constructed by finding the slope at various points along the s-t graph.

V-T GRAPH

Plots of velocity vs. time can be used to find acceleration vs. time curves. Finding the slope of the line tangent to the velocity curve at any point is the acceleration at that point (or \( a = \frac{dv}{dt} \)).

Therefore, the a-t graph can be constructed by finding the slope at various points along the v-t graph.

Also, the distance moved (displacement) of the particle is the area under the v-t graph during time \( \Delta t \).
A-T GRAPH

Given the a-t curve, the change in velocity ($\Delta v$) during a time period is the area under the a-t curve.

So we can construct a v-t graph from an a-t graph if we know the initial velocity of the particle.

A-S GRAPH

A more complex case is presented by the a-s graph. The area under the acceleration versus position curve represents the change in velocity (recall $\int a \, ds = \int v \, dv$).

$$\frac{1}{2} (v_1^2 - v_o^2) = \int_{s_1}^{s_2} a \, ds = \text{area under the a-s graph}$$

This equation can be solved for $v_1$, allowing you to solve for the velocity at a point. By doing this repeatedly, you can create a plot of velocity versus distance.
V-S GRAPH

Another complex case is presented by the v-s graph. By reading the velocity $v$ at a point on the curve and multiplying it by the slope of the curve ($dv/ds$) at this same point, we can obtain the acceleration at that point.

$$ a = v \frac{dv}{ds} $$

Thus, we can obtain a plot of $a$ vs. $s$ from the v-s curve.

EXAMPLE

**Given:** v-t graph for a train moving between two stations

**Find:** a-t graph and s-t graph over this time interval

Think about your plan of attack for the problem!
**EXAMPLE (continued)**

**Solution:** For the first 30 seconds the slope is constant and is equal to:

\[ a_{0-30} = \frac{dv}{dt} = \frac{40}{30} = \frac{4}{3} \text{ m/s}^2 \]

Similarly, \( a_{30-90} = 0 \) and \( a_{90-120} = -\frac{4}{3} \text{ m/s}^2 \)

![Graph showing acceleration vs. time](image)

The area under the v-t graph represents displacement.

\[ \Delta s_{0-30} = \frac{1}{2} (40)(30) = 600 \text{ m} \]

\[ \Delta s_{30-90} = (60)(40) = 2400 \text{ m} \]

\[ \Delta s_{90-120} = \frac{1}{2} (40)(30) = 600 \text{ m} \]
CONCEPT QUIZ

1. If a particle starts from rest and accelerates according to the graph shown, the particle’s velocity at \( t = 20 \text{ s} \) is
   
   A) 200 m/s  
   B) 100 m/s  
   C) 0  
   D) 20 m/s

2. The particle in Problem 1 stops moving at \( t = \) _______.
   
   A) 10 s  
   B) 20 s  
   C) 30 s  
   D) 40 s

GROUP PROBLEM SOLVING

Given: The \( v\)-\( t \) graph shown

Find: The \( a\)-\( t \) graph, average speed, and distance traveled for the 30 s interval

Plan: Find slopes of the curves and draw the \( a\)-\( t \) graph. Find the area under the curve—that is the distance traveled. Finally, calculate average speed (using basic definitions!).
GROUP PROBLEM SOLVING

Solution:
For $0 \leq t \leq 10$  \( a = \frac{dv}{dt} = 0.8 \ t \ m/s^2 \)

For $10 \leq t \leq 30$  \( a = \frac{dv}{dt} = 1 \ m/s^2 \)

\[\begin{array}{c|c|c|c|c|c}
 t(s) & 0 & 10 & 30 \\
 \hline
 a(m/s^2) & 0 & 8 & 1 \\
\end{array}\]

\[\begin{align*}
\Delta s_{0-10} &= \int v \ dt = \frac{1}{3} \cdot .4 \cdot (10)^3 = \frac{400}{3} \ m \\
\Delta s_{10-30} &= \int v \ dt = (0.5)(30)^2 + 30(30) - 0.5(10)^2 - 30(10) \\
&= 1000 \ m \\
s_{0-30} &= 1000 + \frac{400}{3} = 1133.3 \ m \\
v_{avg(0-30)} &= \frac{\text{total distance}}{\text{time}} \\
&= \frac{1133.3}{30} \\
&= 37.78 \ m/s
\end{align*}\]
ATTENTION QUIZ

1. If a car has the velocity curve shown, determine the time \( t \) necessary for the car to travel 100 meters.
   
   A) 8 s  
   B) 4 s  
   C) 10 s  
   D) 6 s

2. Select the correct a-t graph for the velocity curve shown.

C) D)  

CURVILINEAR MOTION: RECTANGULAR COMPONENTS
(Sections 12.4)

Today’s Objectives:
Students will be able to:

a) Describe the motion of a particle traveling along a curved path.

b) Relate kinematic quantities in terms of the rectangular components of the vectors.

In-Class Activities:
- Check homework, if any
- Reading quiz
- Applications
- General curvilinear motion
- Rectangular components of kinematic vectors
- Concept quiz
- Group problem solving
- Attention quiz
READING QUIZ

1. In curvilinear motion, the direction of the instantaneous velocity is always
   A) tangent to the hodograph.
   B) perpendicular to the hodograph.
   C) tangent to the path.
   D) perpendicular to the path.

2. In curvilinear motion, the direction of the instantaneous acceleration is always
   A) tangent to the hodograph.
   B) perpendicular to the hodograph.
   C) tangent to the path.
   D) perpendicular to the path.

APPLICATIONS

The path of motion of each plane in this formation can be tracked with radar and their x, y, and z coordinates (relative to a point on earth) recorded as a function of time.

How can we determine the velocity or acceleration of each plane at any instant?

Should they be the same for each aircraft?
A roller coaster car travels down a fixed, helical path at a constant speed.

How can we determine its position or acceleration at any instant?

If you are designing the track, why is it important to be able to predict the acceleration of the car?

---

A particle moving along a curved path undergoes curvilinear motion. Since the motion is often three-dimensional, vectors are used to describe the motion.

A particle moves along a curve defined by the path function, $s$.

The position of the particle at any instant is designated by the vector $\mathbf{r} = \mathbf{r}(t)$. Both the magnitude and direction of $\mathbf{r}$ may vary with time.

If the particle moves a distance $\Delta s$ along the curve during time interval $\Delta t$, the displacement is determined by vector subtraction: $\Delta \mathbf{r} = \mathbf{r}' - \mathbf{r}$
VELOCITY

Velocity represents the rate of change in the position of a particle.

The average velocity of the particle during the time increment $\Delta t$ is

$$v_{avg} = \frac{\Delta r}{\Delta t}.$$ 

The instantaneous velocity is the time-derivative of position

$$v = \frac{dr}{dt}.$$ 

The velocity vector, $v$, is always tangent to the path of motion.

The magnitude of $v$ is called the speed. Since the arc length $\Delta s$ approaches the magnitude of $\Delta r$ as $t \to 0$, the speed can be obtained by differentiating the path function ($v = ds/dt$). Note that this is not a vector!

ACCELERATION

Acceleration represents the rate of change in the velocity of a particle.

If a particle’s velocity changes from $v$ to $v'$ over a time increment $\Delta t$, the average acceleration during that increment is:

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{(v - v')}{\Delta t}.$$ 

The instantaneous acceleration is the time-derivative of velocity:

$$a = \frac{dv}{dt} = \frac{d^2r}{dt^2}.$$ 

A plot of the locus of points defined by the arrowhead of the velocity vector is called a hodograph. The acceleration vector is tangent to the hodograph, but not, in general, tangent to the path function.
CONCEPT QUIZ

1. If the position of a particle is defined by \( r = [(1.5t^2 + 1) \hat{i} + (4t - 1) \hat{j}] \) (m), its speed at \( t = 1 \) s is
   A) 2 m/s  
   B) 3 m/s  
   C) 5 m/s  
   D) 7 m/s

2. The path of a particle is defined by \( y = 0.5x^2 \). If the component of its velocity along the x-axis at \( x = 2 \) m is \( v_x = 1 \) m/s, its velocity component along the y-axis at this position is
   A) 0.25 m/s  
   B) 0.5 m/s  
   C) 1 m/s  
   D) 2 m/s

ATTENTION QUIZ

1. If a particle has moved from A to B along the circular path in 4s, what is the average velocity of the particle ?
   A) 2.5 \( \hat{i} \) m/s  
   B) 2.5 \( \hat{i} \) +1.25 \( \hat{j} \) m/s  
   C) 1.25 \( \pi \hat{i} \) m/s  
   D) 1.25 \( \pi \hat{j} \) m/s

2. The position of a particle is given as \( r = (4t^2 \hat{i} - 2x \hat{j}) \) m. Determine the particle’s acceleration.
   A) (4 \( \hat{i} +8 \hat{j} \)) m/s\(^2\)  
   B) (8 \( \hat{i} -16 \hat{j} \)) m/s\(^2\)  
   C) (8 \( \hat{i} \)) m/s\(^2\)  
   D) (8 \( \hat{j} \)) m/s\(^2\)
MOTION OF A PROJECTILE (Section 12.6)

**Today’s Objectives:**
Students will be able to
analyze the free-flight motion of a projectile.

**In-Class Activities:**
- Check homework, if any
- Reading quiz
- Applications
- Kinematic equations for projectile motion
- Concept quiz
- Group problem solving
- Attention quiz

**READING QUIZ**

1. The downward acceleration of an object in free-flight motion is
   A) zero  
   B) increasing with time
   C) 9.81 m/s²  
   D) 9.81 cm/s²

2. The horizontal component of velocity remains _________ during a free-flight motion.
   A) zero  
   B) constant
   C) at 9.81 m/s²  
   D) at 10 m/s²
APPLICATIONS

A kicker should know at what angle, $\theta$, and initial velocity, $v_0$, he must kick the ball to make a field goal.

For a given kick “strength”, at what angle should the ball be kicked to get the maximum distance?

APPLICATIONS (continued)

A fireman wishes to know the maximum height on the wall he can project water from the hose. At what angle, $\theta$, should he hold the hose?
CONCEPT OF PROJECTILE MOTION

Projectile motion can be treated as two rectilinear motions, one in the horizontal direction experiencing zero acceleration and the other in the vertical direction experiencing constant acceleration (i.e., gravity).

For illustration, consider the two balls on the left. The red ball falls from rest, whereas the yellow ball is given a horizontal velocity. Each picture in this sequence is taken after the same time interval. Notice both balls are subjected to the same downward acceleration since they remain at the same elevation at any instant. Also, note that the horizontal distance between successive photos of the yellow ball is constant since the velocity in the horizontal direction is constant.

KINEMATIC EQUATIONS: HORIZONTAL MOTION

Since $a_x = 0$, the velocity in the horizontal direction remains constant ($v_x = v_{ox}$) and the position in the $x$ direction can be determined by:

$$x = x_o + (v_{ox})(t)$$

Why is $a_x$ equal to zero (assuming movement through the air)?
KINEMATIC EQUATIONS: VERTICAL MOTION

Since the positive y-axis is directed upward, \( a_y = -g \). Application of the constant acceleration equations yields:

\[ v_y = v_{oy} - g(t) \]
\[ y = y_o + (v_{oy})(t) - \frac{1}{2}g(t)^2 \]
\[ v_y^2 = v_{oy}^2 - 2g(y - y_o) \]

For any given problem, only two of these three equations can be used. Why?

Example 1

**Given:** \( v_o \) and \( \theta \)

**Find:** The equation that defines \( y \) as a function of \( x \).

**Plan:** Eliminate time from the kinematic equations.

**Solution:** Using \( v_x = v_o \cos \theta \) and \( v_y = v_o \sin \theta \)

We can write: \( x = (v_o \cos \theta)t \) or \( t = \frac{x}{v_o \cos \theta} \)

\[ y = (v_o \sin \theta)t - \frac{1}{2} g(t)^2 \]

By substituting for \( t \):

\[ y = (v_o \sin \theta) \left( \frac{x}{v_o \cos \theta} \right) - \left( \frac{g}{2} \right) \left( \frac{x}{v_o \cos \theta} \right)^2 \]
Example 1 (continued):

Simplifying the last equation, we get:

\[ y = (x \tan \theta) - \left( \frac{g x^2}{2v_0^2} \right)(1 + \tan^2 \theta) \]

The above equation is called the “path equation” which describes the path of a particle in projectile motion. The equation shows that the path is parabolic.

Example 2

**Given:** Snowmobile is going 15 m/s at point A.

**Find:** The horizontal distance it travels (R) and the time in the air.

**Solution:**

First, place the coordinate system at point A. Then write the equation for horizontal motion.

\[ \vec{x}_B = \vec{x}_A + \vec{v}_A t_{AB} \quad \text{and} \quad \vec{v}_A = 15 \cos 40^\circ \text{ m/s} \]

Now write a vertical motion equation. Use the distance equation.

\[ \vec{y}_B = \vec{y}_A + \vec{v}_A t_{AB} - 0.5 g t_{AB}^2 \quad \vec{v}_A = 15 \sin 40^\circ \text{ m/s} \]

Note that \( x_B = R, x_A = 0, y_B = -(3/4)R, \) and \( y_A = 0. \)

Solving the two equations together (two unknowns) yields

\( R = 19.0 \text{ m} \quad t_{AB} = 2.48 \text{ s} \)
CONCEPT QUIZ

1. In a projectile motion problem, what is the maximum number of unknowns that can be solved?
   A) 1   B) 2   C) 3   D) 4

2. The time of flight of a projectile, fired over level ground with initial velocity $V_o$ at angle $\theta$, is equal to
   A) $(v_o \sin \theta)/g$   B) $(2v_o \sin \theta)/g$
   C) $(v_o \cos \theta)/g$   D) $(2v_o \cos \theta)/g$

GROUP PROBLEM SOLVING

Given: Skier leaves the ramp at $\theta_A = 25^\circ$ and hits the slope at B.

Find: The skier’s initial speed $v_A$.

Plan: Establish a fixed x,y coordinate system (in the solution here, the origin of the coordinate system is placed at A). Apply the kinematic relations in x and y-directions.
GROUP PROBLEM SOLVING (continued)

Solution:

Motion in x-direction:
Using \( x_B = x_A + v_{ox}(t_{AB}) \)
\[
\begin{align*}
    t_{AB} &= \frac{(4/5)100}{v_A \cos 25} \\
    &= \frac{88.27}{v_A}
\end{align*}
\]

Motion in y-direction:
Using \( y_B = y_A + v_{oy}(t_{AB}) - \frac{1}{2} g(t_{AB})^2 \)
\[
\begin{align*}
    -64 &= 0 + v_A \sin 45 \left( \frac{80}{v_A \cos 25} \right) - \frac{1}{2} (9.81) \left( \frac{88.27}{v_A} \right)^2 \\
    \frac{v_A}{2} &= 19.42 \text{ m/s}
\end{align*}
\]

ATTENTION QUIZ

1. A projectile is given an initial velocity \( v_0 \) at an angle \( \phi \) above the horizontal. The velocity of the projectile when it hits the slope is ______ the initial velocity \( v_0 \).
   A) less than  
   B) equal to  
   C) greater than  
   D) None of the above.

2. A particle has an initial velocity \( v_0 \) at angle \( \theta \) with respect to the horizontal. The maximum height it can reach is when
   A) \( \theta = 30^\circ \)  
   B) \( \theta = 45^\circ \)  
   C) \( \theta = 60^\circ \)  
   D) \( \theta = 90^\circ \)
End of the Lecture

Let Learning Continue